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RESOLUTION SPACE, NETWORKS AND NON-SELF-ADJOINT SPECTRAL THEORY--ETC(U)
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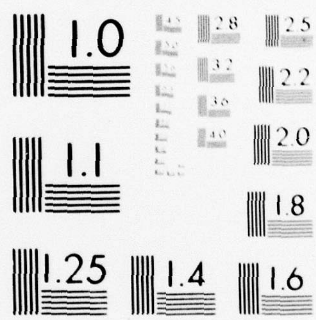

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(6) RESOLUTION SPACE, NETWORKS AND NON-SELF-ADJOINT SPECTRAL THEORY.

by

(10) R. Saeks

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Abstract

Research conducted under the auspices of AFOSR Grant 74-2631 on "Resolution Space, Networks, and Non-Self-Adjoint Spectral Theory" during the period April 1, 1977 through March 31, 1978 is surveyed. Specific attention is given to the solution of a Wiener-Hopf-like "Basic Stochastic Optimization Problem", the solution of the feedback system stability problem for nonlinear and time-varying systems, and the formulation of a theory of systems defined in a Banach resolution space.

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I. Introduction

The theory of operators defined on a resolution space was initiated a decade ago as a mathematical setting in which to formulate a unified theory of linear networks and systems.[11,13] With the aid of recent progress in stability theory[14] and Wiener-Hopf optimization[4,21,22,23], that goal is now in sight. Indeed, it has been surpassed in several areas wherein surprising inroads have been made into the nonlinear theory.[5,14,15]

Our research during the past year may naturally be subdivided into three areas dealing with Wiener-Hopf optimization theory, stability theory, and the developement of the foundations for a theory of operators on a Banach resolution space. The Wiener-Hopf work began with the formulation of a Wiener filter for operators on resolution space[21,22,23] which has since been extended into a "basic stochastic optimization problem" which subsumes most (all?) of the quadradic optimization problems studied in linear system theory.[4] These include the Wiener and Kalman filters, the sevomechanism and optimal regulator problems, the principle of optimality, and the separation principle. The resultant theory

is applicable to time-variant, distributed, and finite time linear systems as well as the classical linear lumped time-invariant systems. It is limited only by the requirement that certain covariance operators have a finite trace and questions surrounding the existence of the "causal part" of an operator.[22]

Our work in the stability area has been centered on the formulation of necessary conditions for the stability of a general nonlinear feedback system.[14] This has been achieved, in part, through the formulation of homology theory for the set of causal invertible operators on a resolution space. Here, our previously developed theorem to the effect that the causal invertibility property is a homotopic invariant[16] is employed to map the set of all causal invertible operators on a resolution space onto an appropriate abelian semigroup in such a manner that the oeprators which admit a causal inverse (and hence define a stable feedback system) are mapped onto the units of the semigroup. By explicitally constructing his homology for various classes of operator we have successfully replicated most of the classical necessary and sufficient conditions for feedback system stability.[14] Moreover, the technique employed appears to be applicable to large classes of time-variant and nonlinear systems; a possibility which is presently being explored.

Our third major area of activity during the past year has been directed toward the extension of the resolution space concept from Hilbert to Banach space. The most spectacular result in this endeavor has been the formulation of a miniphase operator factorization theory for operators defined on a Banach space in which the factor space is a Hilbert space.[23,24] This, in turn, implies that the reproducing kernel resolution space for a (reflexive) Banach space valued random variable is a Hilbert space and that the

scattering variables for a network defined in Banach space take their values in a Hilbert space.[23,24] In other areas a class of Orlicz resolution spaces have been defined which have a "quasi-pythagorian property that allows much of the Hilbert resolution space theory to be extended to an Orlicz resolution space.

II. Wiener-Hopf Theory

Our "basic stochastic optimization problem" is characterized by the block diagram of figure 1.[4] Here, α and β are given stochastic processes,

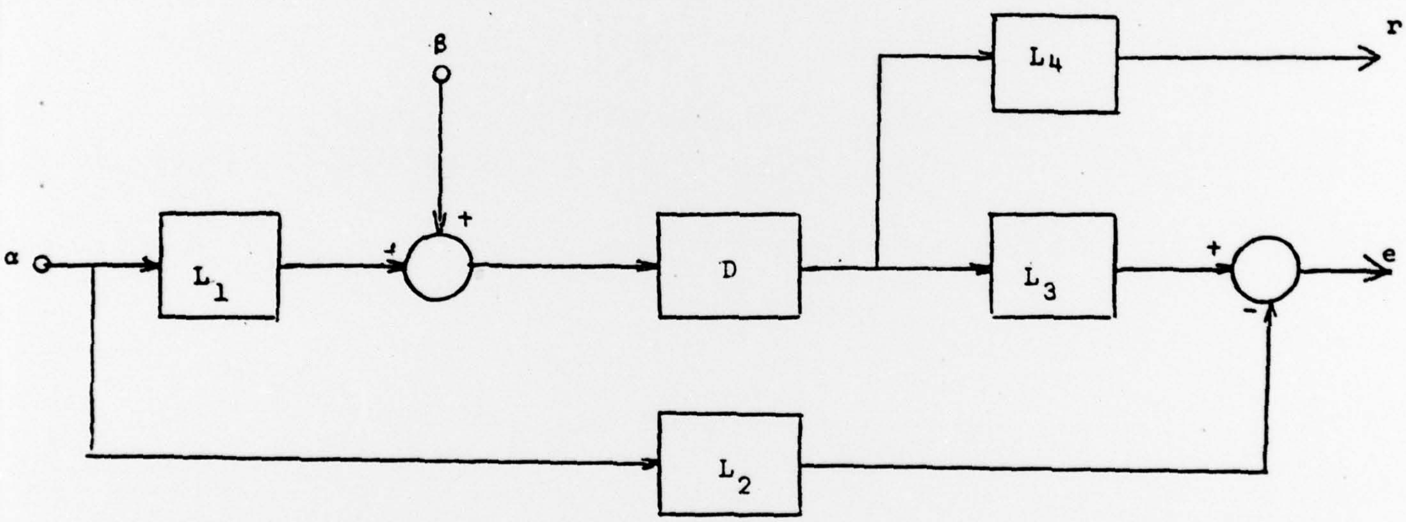

Figure 1. Block diagram for the "basic stochastic optimization problem".

the L_i are (not necessarily causal) operators on a resolution space, and we are required to find a causal operator, D_o, which minimizes

1. $$J(D) = E\{||R||^2 + ||e||^2\}$$

over the set of all causal operators. A solution to this problem is given by

2. $$D_o = M^{-L}\,[M^{*-R}L_3^*(L_2Q_\alpha L_1^* + L_2Q_{\alpha\beta})N^{*-L}]_C N^{-R}$$

where M and N are left and right miniphase factorizations of

3. $$L_3^*L_3^* + L_4^*L_4^* = M^*M$$

and

4. $$L_1Q_\alpha L_1^* + Q_\beta + L_1Q_{\alpha\beta} + Q_{\beta\alpha}L_1^* = NN^*$$

respectively, and $[\ ]_C$ denotes the "causal part" operator. This solution is well defined whenever the "causal part" exists and α, β, and the L_i are restricted in such a way as to guarantee the finiteness of $J(D_o)$.[4]

Interestingly, this single "basic stochastic optimization problem" includes most of the quadradic optimization problems classically studied in system theory. For instance, the restricted Wiener filter derived by Tung[21,22,23] corresponds to the special case where α and β are independent, $L_1 = L_2 = L_3 = 1$, and $L_4 = 0$. A less restricted form of the Wiener filter is obtained by letting $L_3 = 1$ and $L_4 = 0$ with α,β, L_1 and L_2 arbitrary. In particular, if $L_1 = 1$ and $L_2 = P$ (and ideal predictor) this reduces to the classical Wiener predictor.[4] Similarly, by letting the L_i represent the constituant operators which make up a state model a Kalman filter may be obtained. In this case, the recent theorem of Stienberger[20] is used to transform equation 2. into the feedback structure classically associated with the Kalman filter.

To convert the "basic stochastic optimization problem" into a classical open loop control problem, one takes L_4 to be non-zero with our choice of α, β, and the L_i defining the various classical input-output and state space control problems. The resultant open loop controllers may then be used to determine equivalent

feedback controllers via the classical transformation of variables. Of course, one must further verify the stability of the resultant feedback controllers. For the case of a state space feedback controller or an input-output controller with causal open loop gain, this presents no difficulty. For the general case, however, the stability of the resultant feedback control system remains an open question.[4]

The applicability of the solution to the "basic stochastic optimization problem" given by equation 2.` is limited only by the existence of the "causal part" and the requirement that $J(D_o)$ be finite. The former problem was studied several years ago and, unfortunately, proved to be highly ill-posed.[5,6,13,17] The basic difficulty lies with the fact that the "causal part" operator, viewed as a Banach space mapping from the bounded operators to the causal operators, is not closed.[18] Indeed, both the sets of operators which admit a well defined causal part and those which do not admit a causal part, can be shown to be dense in the bounded operators.[6,8] Of course, many sufficient conditions for the existence of a "causal part" are known.[6,7,8] On the other hand, we believe that the finiteness questions surrounding $J(D_o)$ can be resolved. Indeed, a careful inspection of the derivation of equation 2.[4] will reveal that the given solution is "formally valid" without the finiteness restriction.

III. Stability Theory

Our main activity during the past year in the stability area has been the development of a homology theory for the class of causal invertible operators.[14] Basically, this homology is the

abelian[1] semigroup formed by the connected components in the set of causal invertible operators. By invoking our previously derived theorem to the effect that the property of a causal operator having a causal inverse is a homotopic invariant[16] it then follows that a causal operator has a causal inverse if and only if the connected component in which it lies is a unit in the semigroup. Indeed, this result is a virtual tautology. Interestingly, however, we have discovered that for may classes of operator this homology may be computed explicitally yielding an immediate necessary and sufficient conditon for a causal operator to admit a causal inverse. By working with the return difference operator for a feedback system, this, in turn, has permited us to replicate all of the classical necessary and sufficient conditions for feedback system stability. This includes the classical and multivariable Nyquist criteria, the multdimensional Nyquist criterion, and finite dimensional condition, and the stability conditon for systems defined on l_2^+.[14]

Although homology is usually considered to be an extremely abstract concept, it is ideally suited for our application wherein it transforms a problem defined in an infinite dimensional space of

[1]Rigorously, one must "abelianize" this semigroup by factoring out its comutator. In practice, however, the comutator has been trivial in every example thus far constructed.

(possibly) nonlinear operators into an equivalent problem in an abelian semigroup. Indeed, this semigroup has proven to be finitely generated in every example thus far constructed. As such, we believe that the above described homological approach is potentially capable of yielding necessary and sufficient conditions for the stability of large classes of time-variant and nonlinear feedback systems.

Linear time-invariant systems, either in the multivariable or single-input single-output case are characterized by the semigroup of positive integers yielding the classical result to the effect that a causal operator admits a causal inverse if and only if it is homotopic to the identity. (i.e. it lies in the same connected component as the identity.) Since our homology is dependent only on the homotopic equivalence class of an operator this result can be extended immediately to the class of nonlinear and time-variant operators which are homotopic to a linear time-invariant operator.[14] The open questions are: "how big is this class of nonlinear and time-variant operators?" and "how does one compute the equivalence class of such an operator?". Although not universal, we believe that the above described necessary and sufficient conditon for the causal invertibility (i.e. stability) of those nonlinear and time-variant operators which are homotopic to a linear time-variant operator subsumes the existing literature on causal invertibility. Indeed, to our knowledge, every existing sufficient condition for stability (instability) is essentially a proof that the given return difference operator is (is not) homotopic to the identity.

IV. Banach Resolution Space

Our efforts to extend the resolution space concept to Banach spaces have been directed along two lines of inquiry. First, we have developed a causal operator factorization theory for positive self-adjoint operators which map a reflexive Banach space to its dual.[23,24] A little analysis with the classical theory will reveal that this is precisely the class of operators with which one must deal when working with systems defined in Banach space. For instance, the covariance of a random variable taking values in a reflexive Banach space, B, is such an operator as is the hermitian part of a passive impedance.[23,24] The key to this factorization theory is the fact that the given mapping is factored through a Hilbert space rather than a Banach space. When applied to the covariance of a Banach space valued random process, this implies that its reproducing kernel space is Hilbert space rather than a Banach space, whereas the application of this factorization theory to the hermitian part of a passive impedance defined in Banach space yields scattering variables defined in Hilbert space.[23,24]

Although the factorization theory described above is valid in an arbitrary Banach resolution space[1] the extension of classical results related to strict causality and the additive operator decomposition requires additional structure. This is achieved by a subclass of the Banach resoltuion spaces termed Orlicz resolution spaces which have a "pythagorian" property which allows the convergence arguments typically used in Hilbert resolution space

[1]For the purpose of this theory the self-adjointness condition associated with the resolution of the identity used in Hilbert resolution space theory is subsumed by the simple requirement that the resolution of the identity be made up of contractive idempotents.

to be extended to an Orlicz resolution space. As such, virtually the entirety of Hilbert resolution space theory may be extended to an Orlicz resolution space. We, however, believe that this is the largest class of Banach resolution spaces to which the classical theory can be fully extended.

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